Phenomenological and Numerical Studies of Superfluid Helium Dynamics in the Two-Fluid Model

Christine Darve

Mini-Workshop on Thermal Modeling and Thermal Experiments for Accelerator Magnets

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Overview

a) Two-fluid model for Helium II

b) Motivations for numerical modelization

c) Existing 1-D, 2-D and 3-D numerical simulations

d) Governing equations using P, vn, vs and T variables

e) Computing stage

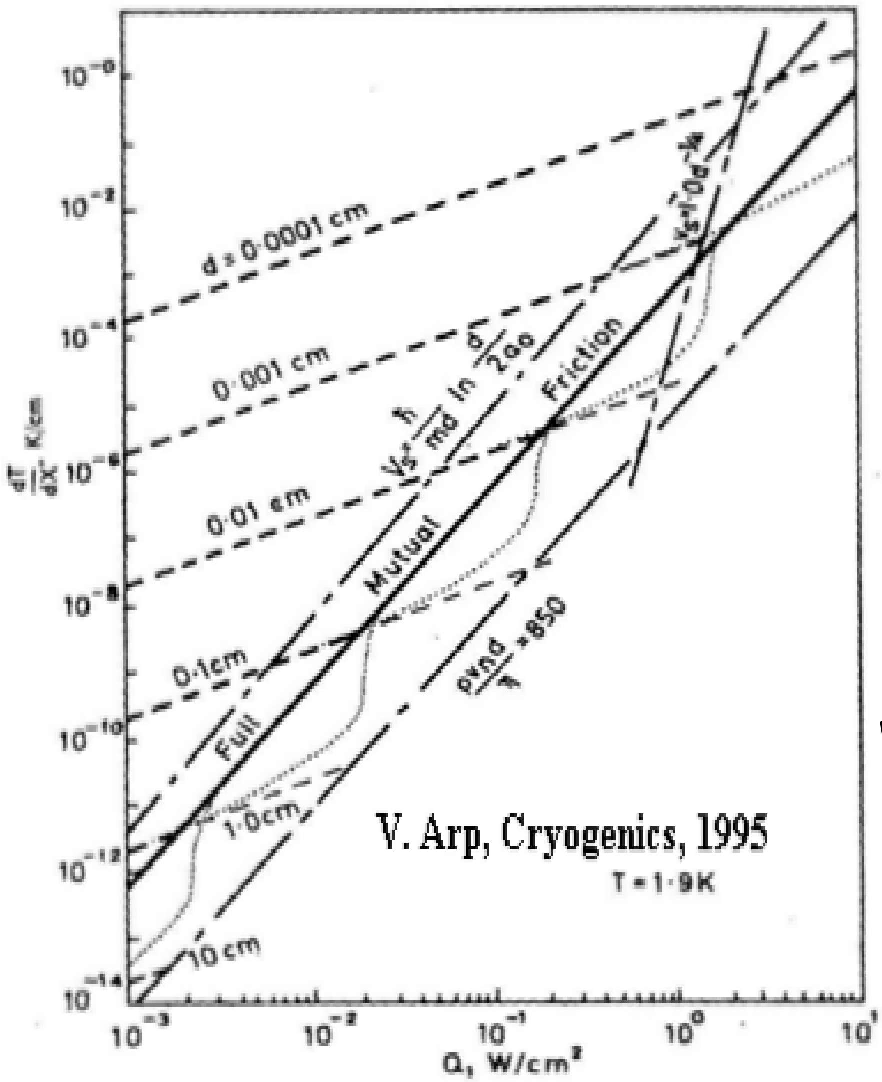
f) Conclusion

Reference papers:

ICEC: Phenomenological approach: a 3-D model of superfluid helium suitable for numerical analysis
by C. Darve, N. A. Patankar, S. W.Van Sciver

LT25: Numerical approach: A method for the three-dimensional numerical simulation of He II
by L. Bottura, C. Darve, N.A. Patankar, S.W. Van Sciver
CERN, Accelerator Technology Department, Geneva, Switzerland
Fermi National Accelerator Laboratory, Accelerator Division, Batavia, IL, USA
Department of Mechanical Engineering, Northwestern University, Evanston, IL, USA
National High Magnetic Laboratory, Florida State University Tallahassee, FL, USA

Two-fluid model for Helium II



The Superfluid fraction:

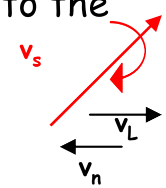
❖Atoms that have undergone BE condensation

❖Finite density, but NO viscosity, carry NO entropy

❖ irrotational behavior for an inviscid fluid $\nabla \times v_s = 0$

❖ vortices can be generated $\nabla T = \frac{\beta \eta_n}{d^2 (\rho s)^2 T} q - \frac{A_{GM} \rho_n}{\rho_s^3 s^4 T^3} q^3$

Differently from classical hydrodynamics, the dissipation in He II vortex motion is NOT due to the viscosity term ν□²v in the NS equation



$$\frac{\partial (\mathbf{v})}{\partial t} + (v \cdot \nabla) v = -\frac{\nabla p}{\rho} + \nu \nabla^2 v$$

✓ Spacing between vortices, δ
where L, is the total length of vortex line per unit volume $\delta \sim \frac{1}{\sqrt{L}}$

✓Order of magnitude (for q = 5,000 W/m², T= 1.8 K)

$$v_n = \frac{\dot{q}}{\rho S T} = 0.035 \frac{\text{m}}{\text{sec}} \qquad v_s = -0.017 \frac{\text{m}}{\text{sec}}$$

✓Line density in steady state conditions $Lo = \left(\frac{\rho_n}{\rho} w \right)^2$

if $L = 10^6 \frac{\text{m}}{\text{m}^3}$ then $\delta \sim 32 \mu\text{m}$

Motivations for numerical modelization

The knowledge of cooling characteristics of He II is indispensable to design superconducting magnets !

Few examples of applications (see introduction talks):

→Thermal counter-flow / Tatsumoto
Fundamental understanding of 2-fluid flow

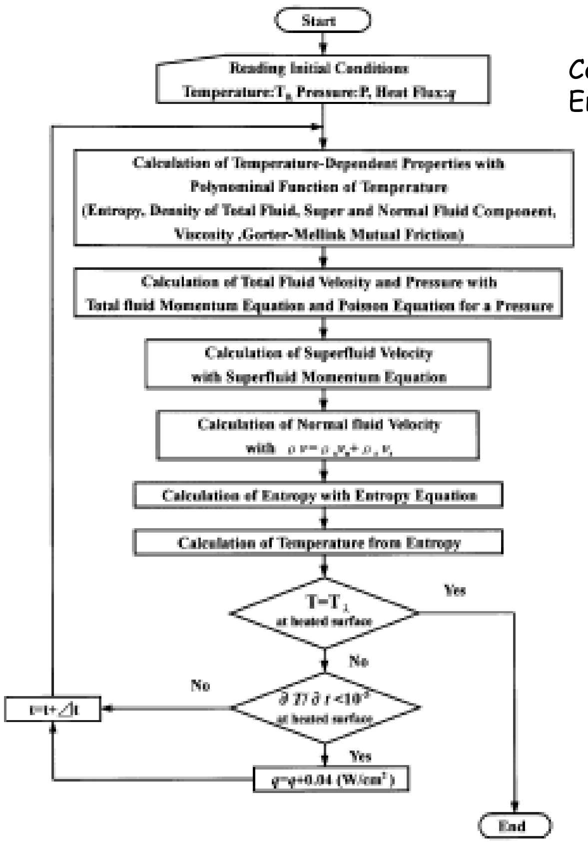
→ Determination of the Critical Heat Flux / Yoshikawa, Shirai
Supraconductor cooling

→ Particle Image Velocimetry technique / Zhang, Fuzier, van Sciver
Effect of Normal and superfluid component

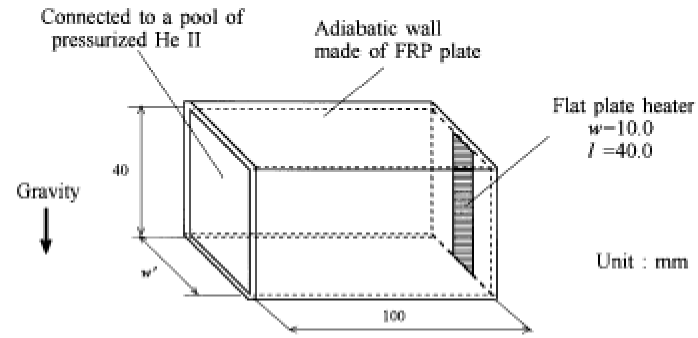
→ 2^{nd} Sound / NHMFL, Fuzier, van Sciver

Thermal counter-flow / Tatsumoto

"Numerical analysis for steady-state two-dimensional heat transfer from a flat plate at one side of a duct containing pressurized He II"
By H. Tatsumoto, K. Fukuda b, M. Shiotsu



Continuity and momentum balances conservation > P
Energy balance conservation Lax algorithm >T



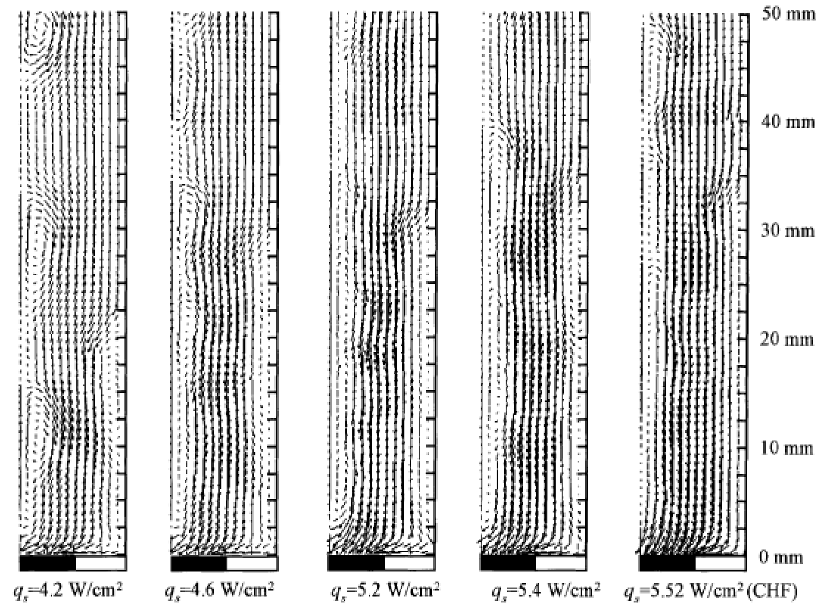


Fig. 6. Velocity distribution of superfluid component in a duct with $A_d/A_h = 2.0$ for bulk temperature of 1.8 K at various heat fluxes and steady-state condition.

Determination of Critical Heat Flux / Yoshikawa, Shirai

"Experiments and 3-D numerical analyses for heat transfer from a flat plate in a duct with contractions filled with liquid He II"
By Yoshikawa K., Shirai Y., Shiotsu M., Hama K.

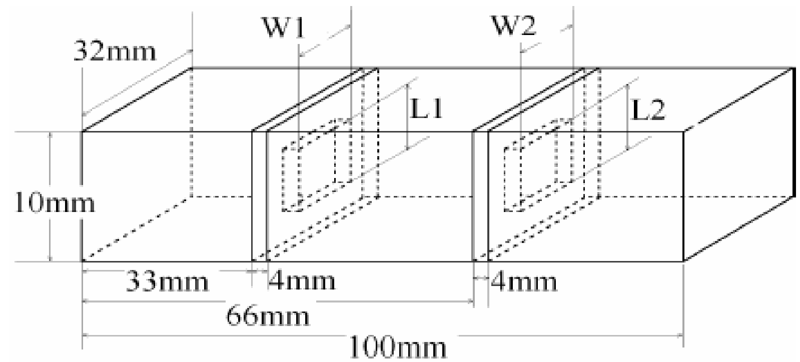


Figure 1 Analytical model of a duct with two contractions

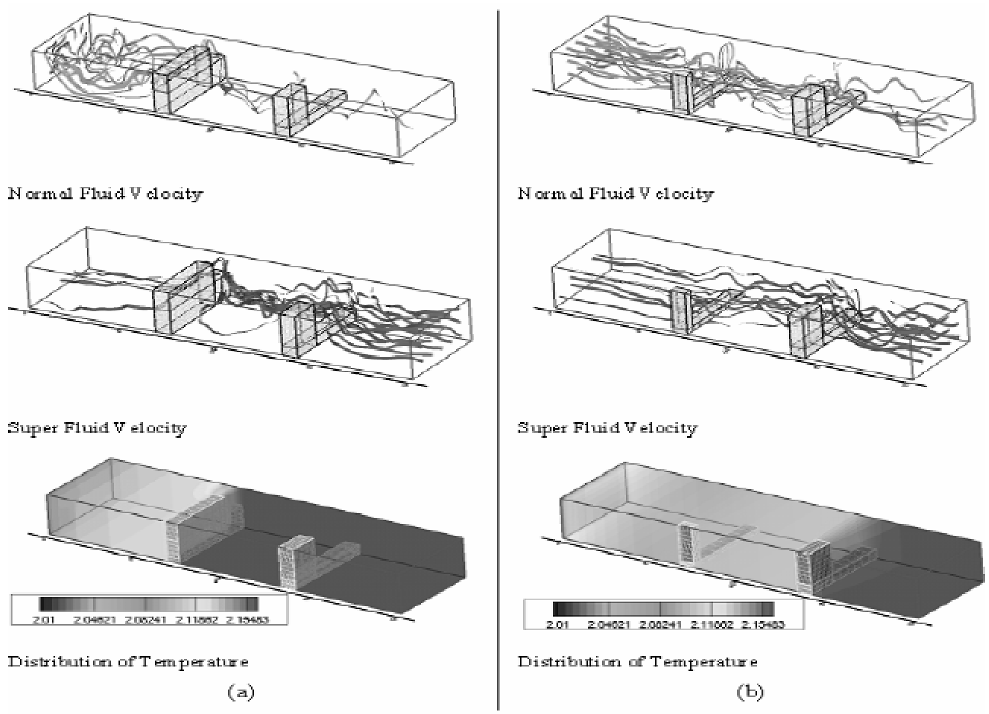


Figure 6 Stream Lines and Distribution of the tempetature

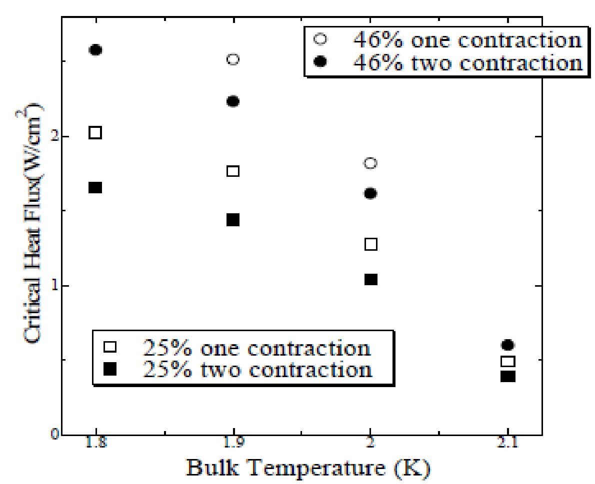


Figure 3 Comparing with CHFs with one contraction and with two contractions

Static and Forced-flow in He II

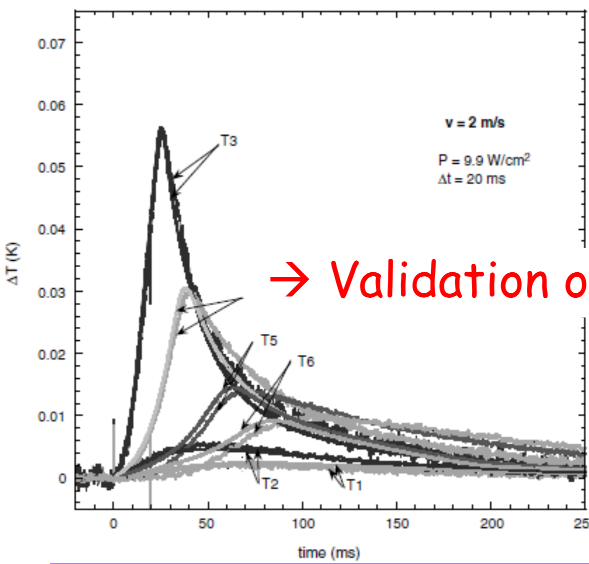
"Experimental measurement and modeling of transient heat transfer in forced flow of He II at high velocities" by S. Fuzier, S.W Van Sciver

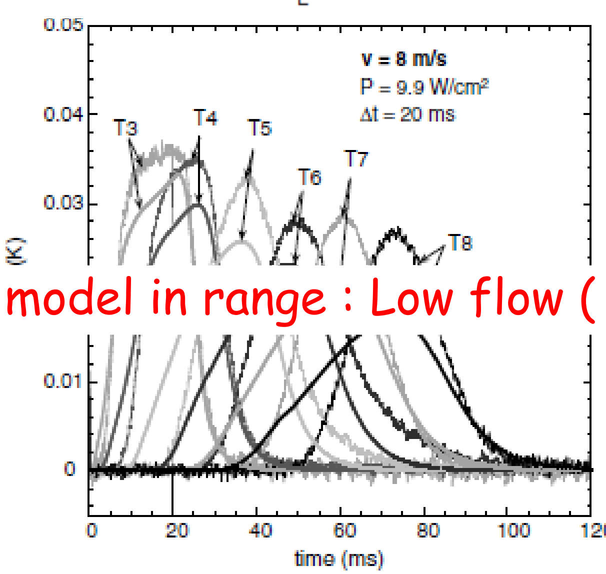
→ Second sound + Modelization of forced convection + counter-flow + pressure effect
→ Forced flow up to 22 m/s
→ Use of high non-linear effective thermal conductivity : keff and Fanning friction factor

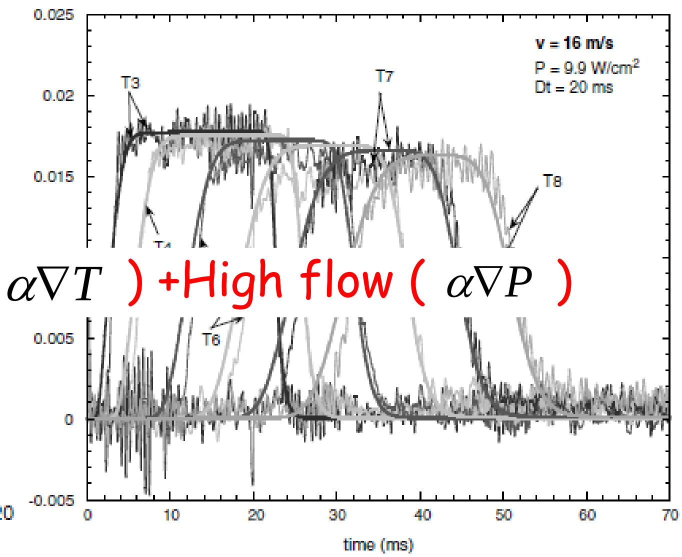
$$\dot{q} = -\left[f^{-1}(T)\frac{dT}{dx}\right]^{1/3} \qquad f(T)=\frac{A_{GM}\cdot\rho_n}{\rho_s^3\cdot s^4 T^3}$$

$$\Rightarrow -\left[f^{-1}(T)\left\{\frac{1}{s\cdot\rho_s}\frac{dP}{dx}+\frac{dT}{dx}\right\}\right]^{1/3}$$

$$\rho C_i^n \frac{T_i^{n+1}-T_i^n}{\Delta t} - \frac{1}{\Delta x}\left[\frac{1}{f_{i+1/2}^{n\frac{1}{3}}\left|\frac{\Delta P}{\rho s_{i+1/2}^n L}+\frac{T_{i+1}^n - T_i^n}{\Delta x}\right|^{\frac{2}{3}}}\left(\frac{\Delta P}{\rho s_{i+1/2}^n L}+\frac{T_{i+1}^{n+1}-T_i^{n+1}}{\Delta x}\right) - \frac{1}{f_{i-1/2}^{n\frac{1}{3}}\left|\frac{\Delta P}{\rho s_{i-1/2}^n L}+\frac{T_i^n-T_{i-1}^n}{\Delta x}\right|^{\frac{2}{3}}}\left(\frac{\Delta P}{\rho s_{i-1/2}^n L}+\frac{T_i^{n+1}-T_{i-1}^{n+1}}{\Delta x}\right)\right] + \rho v C_i^n \frac{T_{i+1}^{n+1}-T_{i-1}^{n+1}}{2\Delta x} = Q_0 + v\frac{\Delta P}{L}$$



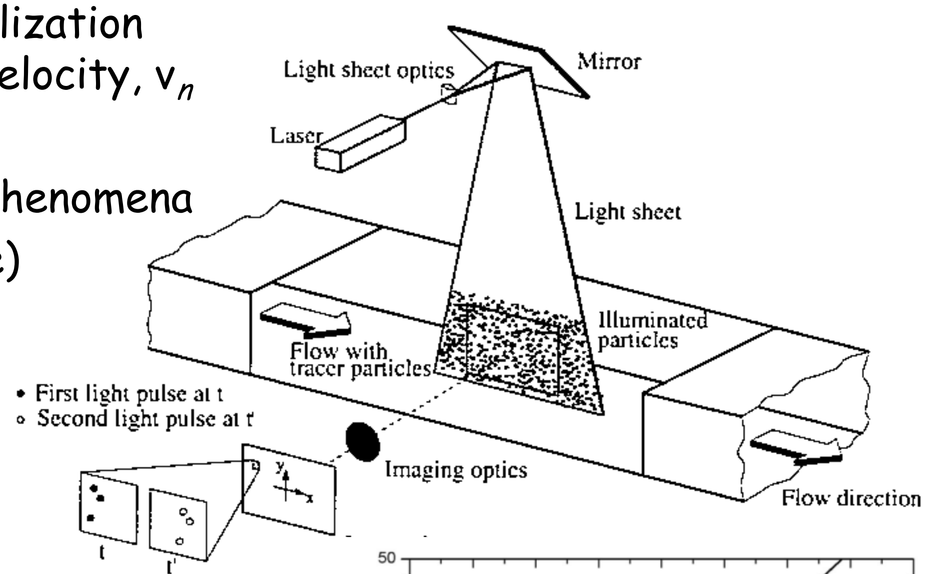




→ Validation of model in range : Low flow ($\alpha\nabla T$) +High flow ($\alpha\nabla P$)

Probing the microscopic scale of He II

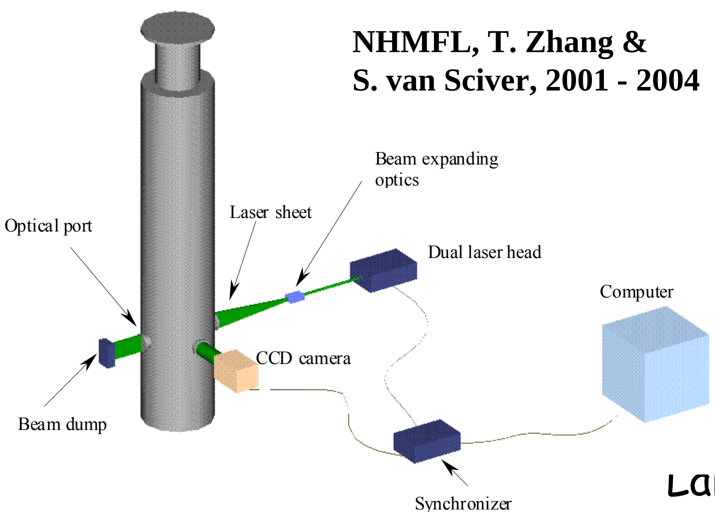
- Particle Image Velocimetry technique:
  - permit 2-D and 3-D flow visualization
  - capable to follow the normal velocity, v_n



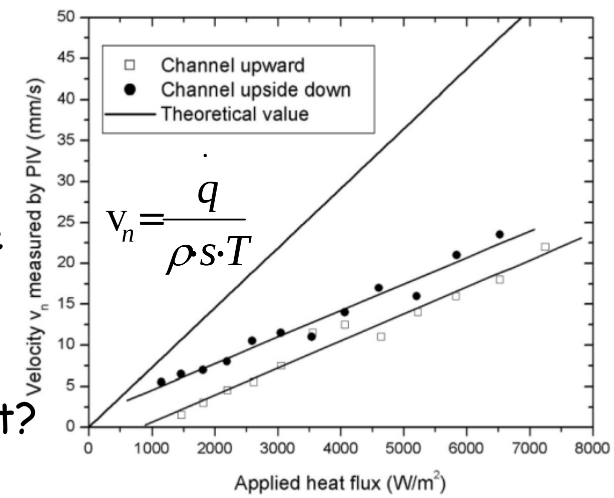
- NHMFL measured internal convection phenomena

Challenges: Particle choices (density, size)

NHMFL, T. Zhang & S. van Sciver, 2001 - 2004



Large discrepancies were observed even with slip velocity correction



Interaction with superfluid component?

→ Need numerical simulation to better understand what happen

Existing 1-D Numerical Simulations

Rao et al. : Forced convection – Steady state and transient (vertical micron-wide GM duct heated at the bottom)

• Method:Finite difference algorithm; 4^{th} order Runge-Kutta, explicit in time

• Variables: Pressure, temperature, normal velocity at BC

• Assumptions: Two-fluid model and the simplified model [Kashani]

$$\dot{q} = \left\{ \rho \cdot Cp \frac{\partial T}{\partial t} + \rho \cdot u \cdot Cp \frac{\partial T}{\partial x} - \frac{\partial}{\partial x} \left\{ \frac{1}{K(T)} \frac{\partial T}{\partial x} \right\}^{1/3} \right\}$$

• Result: Good agreement of both methods with experimental results by Ramada

Bottura et al. : THEA - Simulation of quench propagation

• Method: Finite element algorithm, Taylor-Galerkin, explicit in time

• Variables: Pressure, temperature, velocity

• Assumptions: Use a single-fluid model; add couterflow heat exchange in the energy conservation balance to benchmark

• Result: Good agreement with experimental results by Srinivasan and Hofmann, Kashani et al., Lottin and van Sciver

Existing 2-D Numerical Simulations

Ramadan and Witt: Compared single-fluid and two-fluid models (natural conv. in large He II baths)

- Variables: Pressure, temperature, velocity
- Assumptions: Ignore the thermomechanical effect term and the Gorter-Mellink mutual friction term in the momentum equations for both components
- Result: Illustrate the weakness of the single-fluid model

Tatsumoto: SUPER-2D–Steady state and transient (rectangular duct with varying ratio of heated surface)

- Method: Finite difference, First order upwind scheme, explicit in time
- Variables: Pressure, temperature, heat flux
- Assumptions: Two-fluid model and the energy dissipation based on the mutual friction between the superfluid and normal-fluid components
- Result: Predict the steady state critical heat flux to a precision of about 9 %

Existing 3-D Numerical Simulations

Doi, Shirai, Shiotsu, Yoshikawa – Kyoto : SUPER-3-D, Steady state

[1] "3-D numerical analyses for heat transfer from a flat plate in a duct with contractions filled with pressurized He II".
[2] "Experiments and 3-D numerical analyses for HT from a flat plate in a duct with contractions filled with liquid He II".
(duct w/ 1 and 2 contractions → calculation of Critical Heat Flux)

• Method: Finite difference, First order upwind scheme, explicit in time

Energy balance -> s ->T

Adams-Bashforth method -> vs -> v ->vn

Variables: Pressure, temperature, heat flux , dt=0.5 μsec [1] and dt=2 μsec [2]

• Assumptions: Two-fluid model and the energy dissipation based on the mutual friction between the superfluid and normal-fluid components

• Result: Predict (wrt experiment) the steady state CHF to a precision of about 14 %

• Large memory and time (Parallelized computation using Message passing Interface - MPI)

→ We proposed a different set of equation to ease the calculation of 1-D, 2-D and 3-D structures.

Proposal - PDE (p, v_n, v_s, T)

1. Formulate new and complete Helium II approximations based on the two-fluid model and the theory of GM mutual friction using p, T, v_n and v_s as variables

- Mass, momentum and energy balances conservations permit to derive a partial differential equation (PDE) system of the form:

$$\overline{\overline{\mathbf{m}}}\frac{\partial \mathbf{u}}{\partial t} + \overline{\overline{\mathbf{a}}}\nabla \cdot \mathbf{u} + \nabla \cdot \left(\overline{\overline{\mathbf{g}}}\nabla \cdot \mathbf{u}\right) + \overline{\overline{\mathbf{s}}}\mathbf{u} = \mathbf{q}$$

2. Construct a numerical 1-D then 3-D solver for Helium II based on existing PDE solver

- Calculate shape functions, associated local and global derivatives, jacobian matrix and determinant of 3-D FE (cubic, tetrahedral and wedge)
- Implement the new formulations in 1-D PDE solver for space and time discretization
- Add and modify library for 3-D matrix and vector operations
- Implement a protocol to identify nodes where algebraic and boundary conditions can be imposed

Governing equations for the two-fluid model

Density of He II: $\rho = \rho_n + \rho_s$

Momentum density of He II : $\rho \mathbf{v} = \rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s$

Relative velocity: $\mathbf{w} = \mathbf{v}_n - \mathbf{v}_s$

Thermodynamic potential, Φ : $\Phi = i + \frac{p}{\rho} - sT - \left(\frac{\rho_n}{2\rho}\right) w^2$

Stress tensor $\bar{\bar{\tau}}$:
only depends on the normal fluid

$$\bar{\bar{\tau}} = -\eta \left\{ \nabla^2 \mathbf{v}_n + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}_n) \right\}$$

State variables : p, T $\quad \rho_n = \rho_n(p,T) \quad s = s(p,T) \quad i = i(p,T) \quad k = k(p,T) \quad \eta = \eta(p,T)$

Using thermodynamic properties: $di = \left(\frac{p}{\rho} - \phi C_v T \right) \frac{d\rho}{\rho} + C_v dT \qquad h = i + \frac{p}{\rho}$

$$d\rho = \frac{1+\phi}{c^2} dp - \frac{\phi \rho}{c^2} dh \qquad di = \frac{p}{\rho^2} d\rho + Tds + \frac{w^2}{2} d\left(\frac{\rho_n}{\rho}\right) \qquad d\Phi = \frac{1}{\rho} dp - sdT - \left(\frac{\rho_n}{2\rho}\right) dw^2$$

where ϕ is the Gruneisen parameter, C_v is the specific heat at constant density, *c* is the speed of (first) sound and *h* is the specific enthalpy.

Governing equations – Eq. I and Eq. II

Equation I- Continuity equation & mass balance conservation for normal fluid and superfluid:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n) = m \qquad \frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{v}_s) = -m$$

where m is the rate at which normal fluid is created from superfluid

Equation II and Equation III- Momentum equations

$$\frac{\partial(\rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s)}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n \mathbf{v}_n + \rho_s \mathbf{v}_s \mathbf{v}_s) + \nabla p = -\nabla \cdot \overline{\overline{\tau}} + \rho \mathbf{g}$$

where g is the acceleration of the gravity field

Momentum equation for superfluid [Donnelly] $$\rho_s \frac{\partial \mathbf{v}_s}{\partial t} + \rho_s \mathbf{v}_s \nabla \cdot \mathbf{v}_s + \nabla \Phi = \mathbf{F}_t + \rho_s \mathbf{g}$$

Force associated with turbulence that appears only when the relative velocity between the superfluid and normal fluid components is larger than a critical value

Force of mutual friction is given by GM for counterflow situation $$\mathbf{F}_t = A_{GM} \rho_s \rho_n w^2 \mathbf{w}$$

where A_{GM} is a function of T and, possibly, of w

$$\mathbf{F}_t = Lo \cdot f = \frac{\rho_n^2}{\rho \rho_s^2} \eta_n \rho_s \rho_n w^2 \mathbf{w}$$

Momentum balance conservation – Eq. II & Eq. III

Momentum equation for the normal fluid becomes:

$$\rho_n \frac{\partial \mathbf{v}_n}{\partial t} + \rho_n \mathbf{v}_n \nabla \cdot \mathbf{v}_n + \frac{\rho_n}{\rho} \nabla p + \rho_s s \nabla T + \frac{\rho_s \rho_n}{2\rho} \nabla w^2 = -\nabla \cdot \overline{\overline{\tau}} - \mathbf{F}_t + \rho_n \mathbf{g} - m\mathbf{w}$$

mass exchange

Momentum equation for the superfluid becomes:

acceleration terms

force due to pressure gradient

thermo-mechanical effect

$$\rho_s \frac{\partial \mathbf{v}_s}{\partial t} + \rho_s \mathbf{v}_s \nabla \cdot \mathbf{v}_s + \frac{\rho_s}{\rho} \nabla p - \rho_s s \nabla T - \frac{\rho_s \rho_n}{2\rho} \nabla w^2 = \mathbf{F}_t + \rho_s \mathbf{g}$$

Energy balance conservation – Eq. IV

Equation IV - Internal energy conservation :

$$\frac{\partial \rho s}{\partial t} + \nabla\left(\rho s \mathbf{v}_n\right) = 0$$

(A) Irreversible motion the entropy is conserved

$$\frac{\partial}{\partial t}\left(\rho i + \frac{\rho_n \mathbf{v}_n^2}{2} + \frac{\rho_s \mathbf{v}_s^2}{2}\right) + \nabla\cdot\left(\rho i\mathbf{v} + \frac{\rho_n \mathbf{v}_n^2}{2}\mathbf{v}_n + \frac{\rho_s \mathbf{v}_s^2}{2}\mathbf{v}_s\right) + \nabla p\mathbf{v} + \nabla\rho_s sT\mathbf{w} + \nabla\frac{\rho_s\rho_n}{2\rho}w^2\mathbf{w} - \nabla\left(k\nabla T\right) = -\nabla\cdot\left(\bar{\bar{\tau}}\mathbf{v}_n\right) + \rho\mathbf{g}\mathbf{v} + q$$

We can express the kinetic energy as

(B)

$$\frac{\partial}{\partial t}\left(\frac{\rho_n \mathbf{v}_n^2}{2} + \frac{\rho_s \mathbf{v}_s^2}{2}\right) + \nabla\cdot\left(\frac{\rho_n \mathbf{v}_n^2}{2}\mathbf{v}_n + \frac{\rho_s \mathbf{v}_s^2}{2}\mathbf{v}_s\right) + \mathbf{v}\nabla p + \rho_s s\mathbf{w}\nabla T + \frac{\rho_s\rho_n}{2\rho}\mathbf{w}\nabla w^2 + \mathbf{F}_t\mathbf{w} = -\mathbf{v}_n\nabla\cdot\bar{\bar{\tau}} + \rho\mathbf{g}\mathbf{v} - \frac{m}{2}w^2$$

(A)-(B)

$$\frac{\partial(\rho i)}{\partial t} + \nabla\cdot\left(\rho i\mathbf{v}\right) + p\nabla\mathbf{v} + T\nabla\rho_s s\mathbf{w} + w^2\nabla\frac{\rho_s\rho_n}{2\rho}\mathbf{w} - \mathbf{F}_t\mathbf{w} - \nabla\cdot\left(k\nabla T\right) = -\bar{\bar{\tau}}\cdot\nabla\cdot\mathbf{v}_n + q$$

$$\rho\frac{\partial i}{\partial t} + \rho\mathbf{v}\nabla\cdot i + p\nabla\mathbf{v} + T\nabla\rho_s s\mathbf{w} + w^2\left(\nabla\frac{\rho_s\rho_n}{2\rho}\mathbf{w} - \frac{m}{2}\right) - \mathbf{F}_t\mathbf{w} - \nabla\cdot\left(k\nabla T\right) = -\bar{\bar{\tau}}\cdot\nabla\cdot\mathbf{v}_n + q$$

internal heat convection through entropy transport

represents the internal energy dissipation associated with turbulence, see later…

originates from the transformation of superfluid into normal fluid and vice versa

Substitutions and assumptions

- Friction force is given by GM for counterflow situation: $\mathbf{F}_t = A_{GM}\rho_s\rho_n w^2\mathbf{w}$

  where A_{GM} is a function of T and, possibly, of w

- The divergence of the total velocity is computed through the chain relation:

$$\nabla\cdot\mathbf{v} = \nabla\cdot\left(\frac{\rho_n}{\rho}\mathbf{v}_n + \frac{\rho_s}{\rho}\mathbf{v}_s\right) = \frac{\rho_n}{\rho}\nabla\cdot\mathbf{v}_n + \frac{\rho_s}{\rho}\nabla\cdot\mathbf{v}_s + \mathbf{v}_n\nabla\frac{\rho_n}{\rho} + \mathbf{v}_s\nabla\frac{\rho_s}{\rho}$$

Note that the normal and superfluid velocities appear explicitly

- Contributions related explicitly to the mass exchange ***m*** are small when compared to other terms -> drop them from the balances

- Energy dissipated by viscous dissipation is small compared to other sources of heat transport (e.g. mutual friction) -> treated as a source perturbation

- Terms containing differentials of quantities other than variables (p, $\mathbf{v}_n$, $\mathbf{v}_s$, T) perturbations with respect to the leading terms of the equations

- Variations of the Gruneisen parameter are small

$$\phi\nabla\left(k\nabla T\right) \approx \nabla\left(\phi k\nabla T\right)$$

PDE (p, v_n, v_s, T) form – Continuity & Energy (Eq. I & IV)

convection of the mass

$$\frac{\partial p}{\partial t} + \frac{\rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s}{\rho}\nabla p + \rho_n c^2 \nabla\cdot\mathbf{v}_n + \phi w^2 \frac{\rho_s \rho_n}{2\rho}\nabla\cdot\mathbf{v}_n + \phi T \rho_s s \nabla\cdot\mathbf{v}_n + \rho_s c^2 \nabla\cdot\mathbf{v}_s -$$

1st sound- decompression of ❒

associated to S exchange

mass exchange n<->s

$$\phi w^2 \frac{\rho_s \rho_n}{2\rho}\nabla\cdot\mathbf{v}_s - \phi T \rho_s s \nabla\cdot\mathbf{v}_s - \nabla\cdot\left(\phi k \nabla T\right) - \phi A \rho_s \rho_n w^2 \mathbf{w}\mathbf{v}_n + \phi A \rho_s \rho_n w^2 \mathbf{w}\mathbf{v}_s =$$

generated by viscosity — conduction — due to mutual friction

$$\phi q - \phi\overline{\overline{\tau}}\cdot\nabla\cdot\mathbf{v}_n - \rho c^2 \mathbf{v}_n \nabla\frac{\rho_n}{\rho} - \rho c^2 \mathbf{v}_s \nabla\frac{\rho_s}{\rho} - \phi T \mathbf{w}\nabla\rho_s s - \phi w^2 \mathbf{w}\nabla\frac{\rho_s\rho_n}{2\rho}$$

effect of exp/comp

$$\rho C_v \frac{\partial T}{\partial t} + \rho_n \phi C_v T \nabla\cdot\mathbf{v}_n + w^2 \frac{\rho_s\rho_n}{2\rho}\nabla\cdot\mathbf{v}_n + T\rho_s s \nabla\cdot\mathbf{v}_n + \rho_s \phi C_v T \nabla\cdot\mathbf{v}_s - w^2 \frac{\rho_s\rho_n}{2\rho}\nabla\cdot\mathbf{v}_s -$$

thermal capacity — entropy change — effect of energy exchange n<-> s

$$T\rho_s s \nabla\cdot\mathbf{v}_s + \rho C_v \frac{\rho_n \mathbf{v}_n + \rho_s \mathbf{v}_s}{\rho}\nabla T - \nabla\cdot\left(k\nabla T\right) - A\rho_s\rho_n w^2 \mathbf{w}\mathbf{v}_n + A\rho_s\rho_n w^2 \mathbf{w}\mathbf{v}_s =$$

convection — conduction — energy dissipation due to GM

$$q - \overline{\overline{\tau}}\cdot\nabla\cdot\mathbf{v} - \rho\phi C_v T \mathbf{v}_n \nabla\frac{\rho_n}{\rho} - \rho\phi C_v T \mathbf{v}_s \nabla\frac{\rho_s}{\rho} - T\mathbf{w}\nabla\rho_s s - w^2 \mathbf{w}\nabla\frac{\rho_s\rho_n}{2\rho}$$

external heat — viscous dissipation

PDE (p, v_n, v_s, T) form – Momentum (Eq. II & III)

$$\rho_n \frac{\partial \mathbf{v}_n}{\partial t} + \frac{\rho_n}{\rho}\nabla p + \rho_n \mathbf{v}_n \nabla \cdot \mathbf{v}_n + \frac{\rho_s \rho_n}{\rho}\mathbf{w}\nabla \cdot \mathbf{v}_n - \frac{\rho_s \rho_n}{\rho}\mathbf{w}\nabla \cdot \mathbf{v}_s + \rho_s s \nabla T + A\rho_s \rho_n w^2 \mathbf{v}_n - A\rho_s \rho_n w^2 \mathbf{v}_s = -\nabla \cdot \bar{\bar{\tau}} + \rho_n \mathbf{g}$$

acc. mass

force due to variation of pressure

transport of momentum

mass exchange momentum normal<-> superfluid

thermomechanical effect

mutual friction

viscous effect

gravity effect

$$\rho_s \frac{\partial \mathbf{v}_s}{\partial t} + \frac{\rho_s}{\rho}\nabla p - \frac{\rho_s \rho_n}{\rho}\mathbf{w}\nabla \cdot \mathbf{v}_n + \rho_s \mathbf{v}_s \nabla \cdot \mathbf{v}_s + \frac{\rho_s \rho_n}{\rho}\mathbf{w}\nabla \cdot \mathbf{v}_s - \rho_s s \nabla T - A\rho_s \rho_n w^2 \mathbf{v}_n + A\rho_s \rho_n w^2 \mathbf{v}_s = \rho_s \mathbf{g}$$

Numerical Formulations and THEA

THEA: commercial code by CryoSoft, 1-D Thermal, Hydraulic and Electric Analysis of superconducting cables

use parts of THEA capable of solving generic partial differential equations in a 1-D system of the form

$$\bar{\bar{\mathbf{m}}}\frac{\partial \mathbf{u}}{\partial t} + \bar{\bar{\mathbf{a}}}\nabla\cdot\mathbf{u} + \nabla\cdot\left(\bar{\bar{\mathbf{g}}}\nabla\cdot\mathbf{u}\right) + \bar{\bar{\mathbf{s}}}\mathbf{u} = \mathbf{q}$$

forcing vector

mass matrix

advection matrix

diffusion matrix

source matrix

Write the PDE system as a weighted residual at the nodes with identical weight and shape functions to obtain the system of ODE with discretized matrices

$$M\frac{\partial U}{\partial t} + (A + G - S)U = Q$$

where

$$\mathrm{M}_{\mathrm{IJ}} = \int_{\Omega J} \mathrm{N}_{\mathrm{I}}\mathrm{m}_{\mathrm{IJ}}\mathrm{N}_{\mathrm{J}}d\Omega \qquad G_{IJ} = \int_{\Omega J} \nabla N_I g_{IJ} \nabla N_J d\Omega$$

$$\mathrm{A}_{\mathrm{IJ}} = \int_{\Omega J} \mathrm{N}_{\mathrm{I}}\mathrm{a}_{\mathrm{IJ}}\nabla\mathrm{N}_{\mathrm{J}}d\Omega \qquad \mathrm{S}_{\mathrm{IJ}} = \int_{\Omega J} \mathrm{N}_{\mathrm{I}}\mathrm{s}_{\mathrm{IJ}}\mathrm{N}_{\mathrm{J}}d\Omega$$

$$Q_J = \int_{\Omega J} \nabla N_J q_J d\Omega$$

PDE Solution

THEA solves for each component defined by the user a set of partial differential equations (PDEs), coupled among components whenever chosen by the user, and obtains at any time required the distribution in space, along the conductor length, for the state variable(s). The solution satisfies the initial conditions chosen and the boundary conditions set by the user.

To solve the system of PDEs, THEA uses independent space and time discretization. The space discretization is based on the finite element method, and uses 1-D lagrangian elements with at most fifth order shape functions. The initial mesh is automatically adapted in time to achieve the following objectives:

- track discontinuities such as quench propagating fronts, or lambda transitions in the case of superfluid helium hydrodynamics;
- achieve a user-defined interpolation error on any state variable;
- maintain the element size between maximum and minimum user-defined values.

The user can control the meshing process through the choice of element order and of parameters that affect adaptivity. The time discretization is based on a multi-step finite difference algorithm of the Beam and Warming family with at most third order accuracy. The time step is adapted automatically to achieve a user-defined error, either using a predictive or an a-posteriori error estimate. The user has control on the time integration accuracy through the choice of algorithm, while the time adaptivity is controlled specifying the error estimator and the desired accuracy.

Computing phase

Fortran 77

Submit jobs to Fermilab Farm using CONDOR

Submit jobs to the GRID

Advantages:

User defined environment

Submit jobs in parallel

Inconvenient:

Difficult process to investigate the code instabilities !

Need to use Linux (dual boot machine) or Cygwin

Interactive graphical data analysis programs: PAW

Visualization means are limited: Tecplot

Not User friendly at all !

Verification of the 3-D code on a scalar problem

We first consider a scalar problem with one degree of freedom (Temperature) in a 3-D space (see topologies)

$$\rho C p \frac{\partial T}{\partial t} - \nabla\left(k \nabla T\right) = Q$$

This problem is a typical parabolic equation in time, in the 3-D space and can be tested against analytic results of a 1-D problem

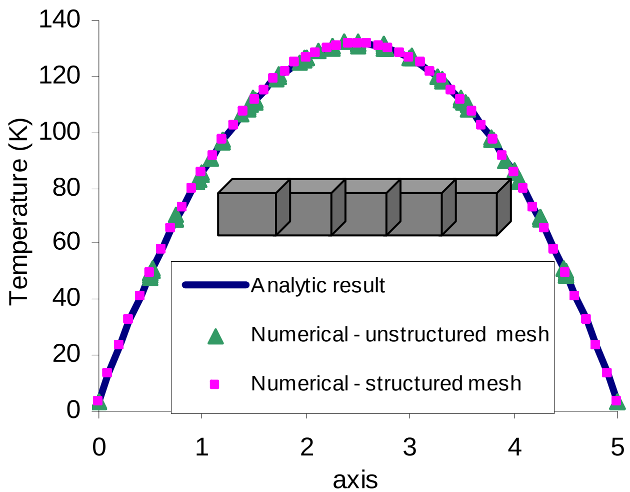
ODE to solve $$M \frac{\partial U}{\partial t} + (A + G - S)U = Q$$

$$U = [T]$$

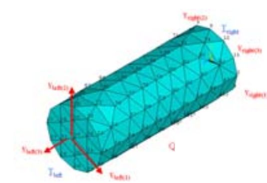
$$\overline{\overline{\mathrm{m}}} = [\rho \cdot Cp] \qquad \overline{\overline{a_x}} = [0] \qquad \overline{\overline{g}} = [k]$$

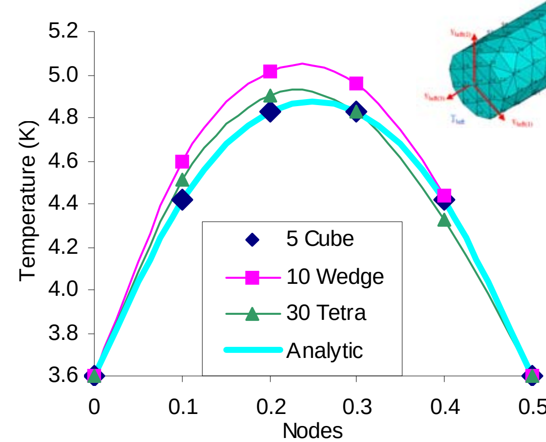
$$\overline{\overline{a_y}} = [0] \qquad \overline{\overline{s}} = [0]$$

$$\overline{\overline{a_z}} = [0] \qquad \overline{\overline{q}} = [q]$$



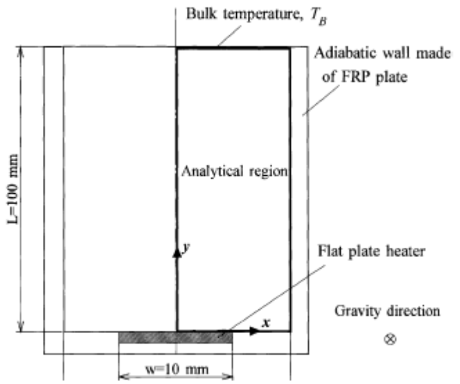
Scalar problem - conduction in steady state (Model A and B, Case II)

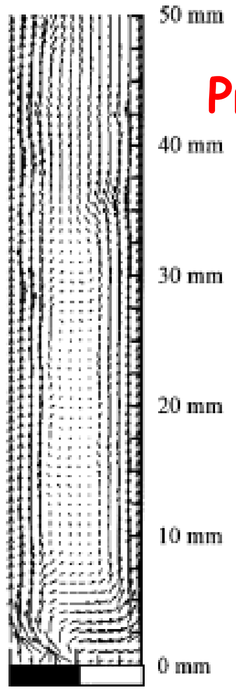




Influence of element for a steady state problem (Model A', Case II)

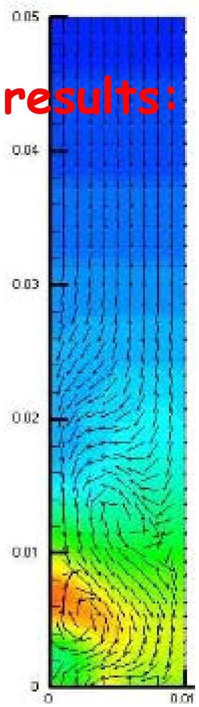
Some results using THEA - PDESolver



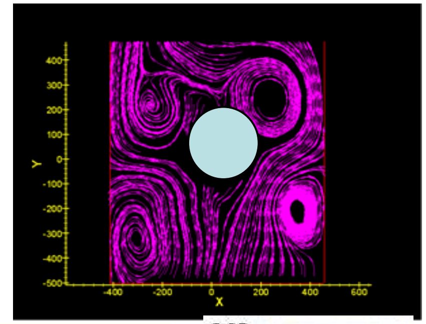


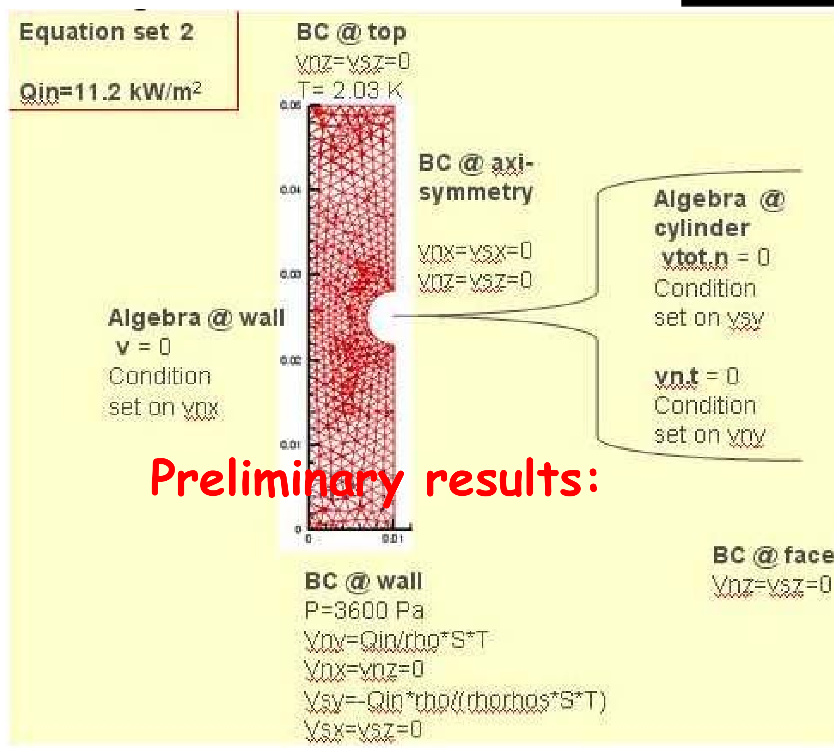
q_s=5.52 W/cm² (CHF)

Preliminary results:

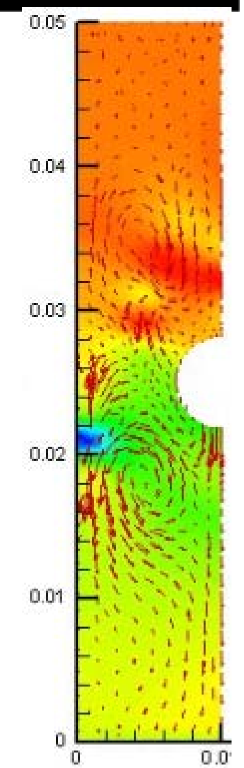


"PIV Measurements of He II Counterflow Around a Cylinder"
By S. Fuzier, S. W. Van Sciver, and T. Zhang





Preliminary results:



Computing Using COMSOL - example

2-D Simulation using asymmetric conditions - Application modes

- Weakly compressible NS → vn, P
- Convection and conduction → T
- Weakly compressible NS → vs, P

Advantages:

Very user friendly !

Possibility to modify governing equation

Add coupling between variables

Use integrated numerical stabilization for normal fluid

Schemes are helpful to stabilize the solution without changing the solution too much → Artificial diffusion (overdamping)

Inconvenient of Physical parameterization:

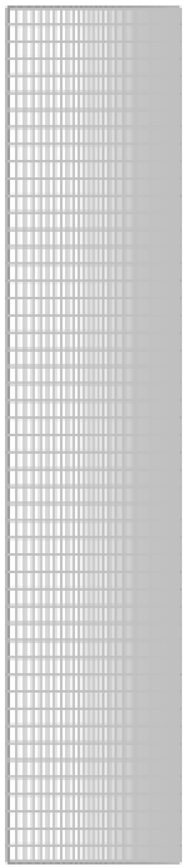
Adapt the governing equation for the superfluid behavior

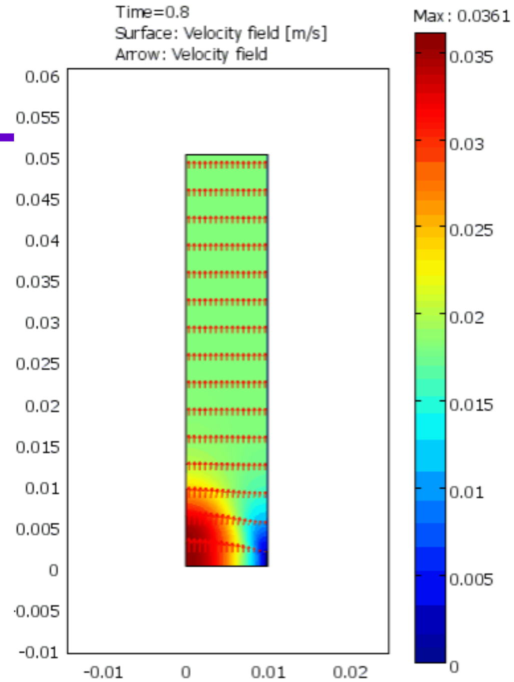
Viscosity = 0 → instability ; work with artificial diffusion

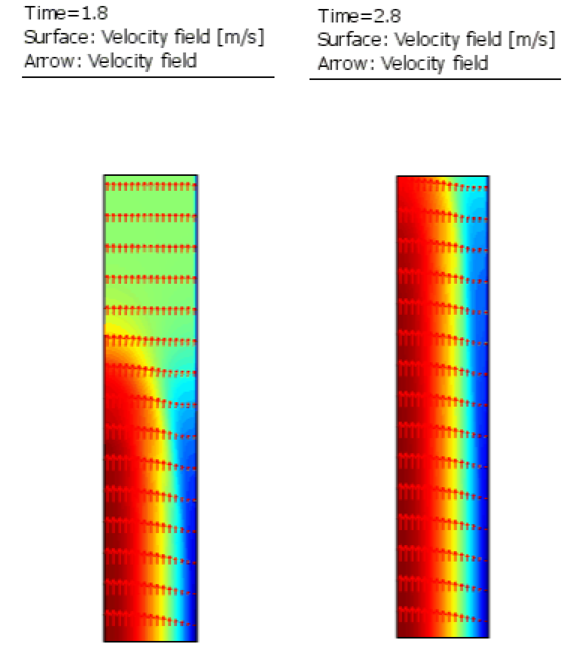
Using COMSOL

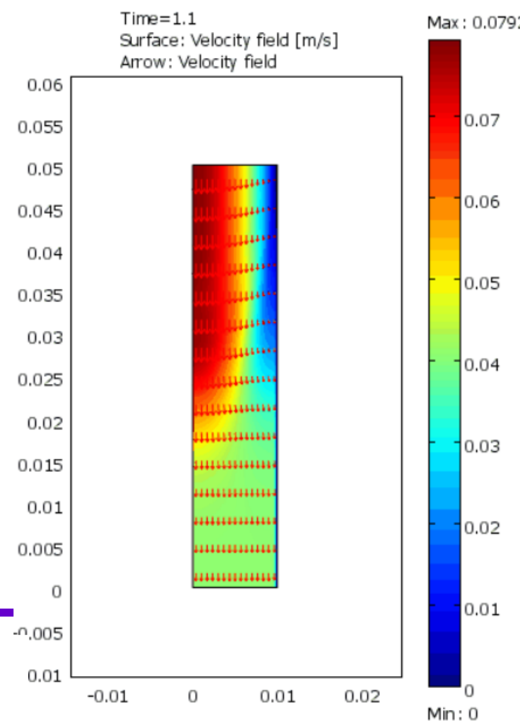
Based on Tatsumoto's example

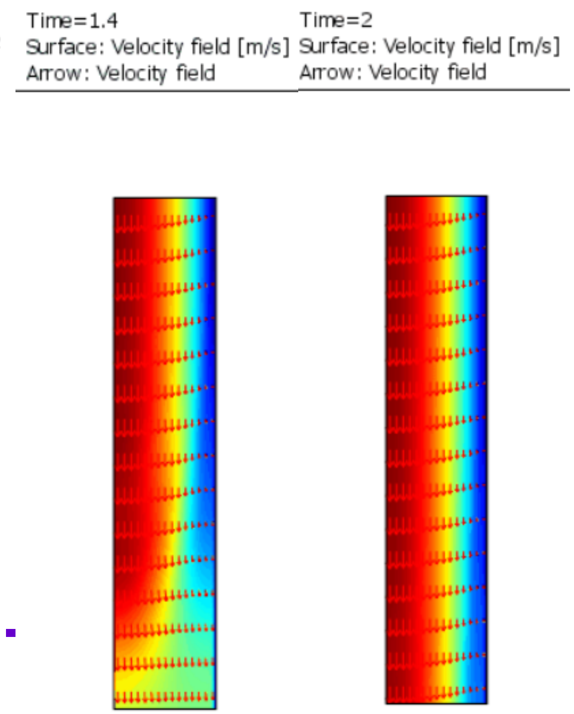
Number of degrees of freedom	56207
Number of mesh points	2601
Number of elements	2500
Triangular	0
Quadrilateral	2500
Number of boundary elements	200
Number of vertex elements	4
Minimum element quality	0.101
Element area ratio	0.1





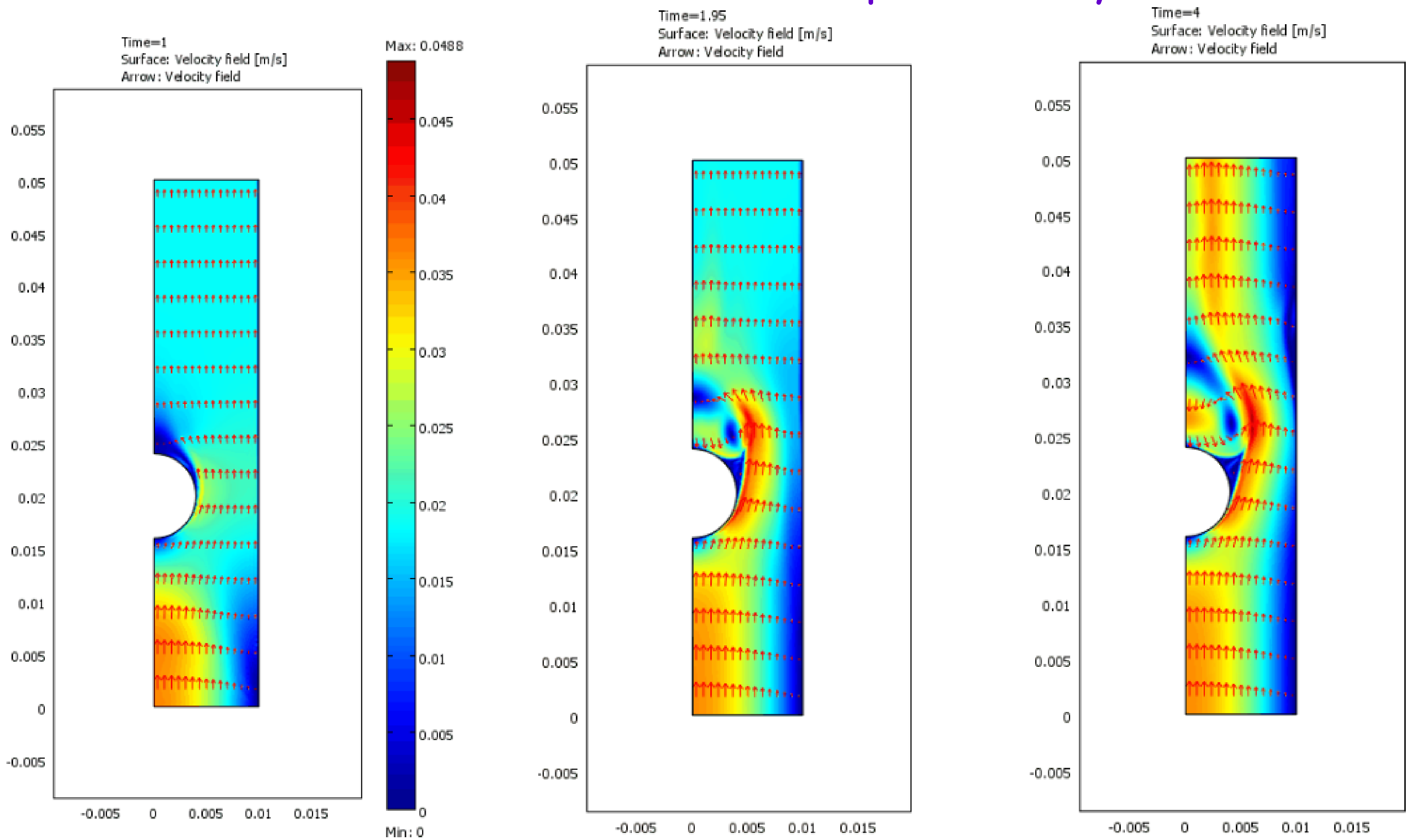






Using COMSOL - Based on NHMFL's example

Normal component velocity



Conclusion

New materials:

- o A new set of equation to validate
- o COMSOL or others to validate this PDE

Remains opened questions in Superfluid Helium behavior Physics:

- PIV: factor 2 between theoretical and numerical model
- S. Fuzier's model to understand: issue with medium range ~ 8 m/s coefficient

Question: how to better understand the phenomenology of 2-fluid flow

One answer: by simulation approach

Challenges:

- o Add coupling, which can introduces inherent physical stabilisation..
- o Non-linearity of superfluid component

Keys to successful numerical simulations:

- o limit the computing time and complexity: CPU used
- o Use a user friendly visualization tool

Weakly Compressible Navier-Stokes

The Weakly Compressible Navier-Stokes application mode allows you to simulate flows where the density of the fluid varies, for instance, as a function of temperature or composition. You select the application mode from the Flow with Variable Density folder in the Model Navigator.

The Weakly Compressible Navier-Stokes application mode contains the fully compressible formulation of the continuity equation and momentum equations:

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$
$$\rho\frac{\partial\mathbf{u}}{\partial t} + \rho\mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \nabla\cdot\left(\eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T) - \left(\frac{2}{3}\eta - \kappa_{dv}\right)(\nabla\cdot\mathbf{u})\mathbf{I}\right) + \mathbf{F} \tag{5-54}$$

The stress tensor used in Equation 5-54 describes a Newtonian fluid, with an added term κ_{dv}. This term expresses the deviation from Stokes' assumption, which states that the fluid particles are in thermodynamic equilibrium with their neighbors. It is very rare that a fluid shows a significant deviation from Stokes' assumption, and κ_{dv} is therefore by default set to zero.

Note: Note that the Weakly Compressible Navier-Stokes application was designed solely to model flow with variable density, where a second application mode—typically a heat transfer application mode—controls the density. This coupling is automatically set up by the Non-Isothermal Flow predefined multiphysics coupling (see "Turbulent Non-Isothermal Flow" on page 270).

Extra – slide : Helium II – PDE simplification

PDE system to solve

and preliminary simplification
To implement in the code

$$\bar{\bar{\mathbf{m}}}\frac{\partial \mathbf{u}}{\partial t}+\bar{\bar{\mathbf{a}}}\nabla\cdot\mathbf{u}+\nabla\cdot\left(\bar{\bar{\mathbf{g}}}\nabla\cdot\mathbf{u}\right)+\bar{\bar{\mathbf{s}}}\mathbf{u}=\mathbf{q}$$

$$\bar{\bar{\mathbf{m}}}=\begin{bmatrix}1&0&0&0\\0&\rho_n&0&0\\0&0&\rho_s&0\\0&0&0&\rho C_v\end{bmatrix}\qquad \bar{\bar{\mathbf{g}}}=\begin{bmatrix}0&0&0&\phi k\\0&0&0&0\\0&0&0&0\\0&0&0&k\end{bmatrix}$$

Vector of unknowns:

$$\mathbf{u}=\begin{bmatrix}p\\ \mathbf{v}_n\\ \mathbf{v}_s\\ T\end{bmatrix}$$

$$\bar{\bar{\mathbf{a}}}=\begin{bmatrix}\frac{\rho_n\mathbf{v}_n+\rho_s\mathbf{v}_s}{\rho} & \rho_n c^2+\phi T\rho_s s+\phi w^2\frac{\rho_s\rho_n}{2\rho} & \rho_s c^2-\phi T\rho_s s-\phi w^2\frac{\rho_s\rho_n}{2\rho} & 0\\ \frac{\rho_n}{\rho} & \rho_n\mathbf{v}_n+\frac{\rho_s\rho_n}{\rho}\mathbf{w} & -\frac{\rho_s\rho_n}{\rho}\mathbf{w} & \rho_s s\\ \frac{\rho_s}{\rho} & -\frac{\rho_s\rho_n}{\rho}\mathbf{w} & \rho_s\mathbf{v}_s+\frac{\rho_s\rho_n}{\rho}\mathbf{w} & -\rho_s s\\ 0 & \rho_n\phi C_v T+T\rho_s s+w^2\frac{\rho_s\rho_n}{2\rho} & \rho_s\phi C_v T-T\rho_s s-w^2\frac{\rho_s\rho_n}{2\rho} & \left(\rho_n\mathbf{v}_n+\rho_s\mathbf{v}_s\right)C_v\end{bmatrix}$$

$$\bar{\bar{\mathbf{s}}}=\begin{bmatrix}0&-\phi A\rho_s\rho_n w^2\mathbf{w}&\phi A\rho_s\rho_n w^2\mathbf{w}&0\\0&A\rho_s\rho_n w^2&-A\rho_s\rho_n w^2&0\\0&-A\rho_s\rho_n w^2&A\rho_s\rho_n w^2&0\\0&-A\rho_s\rho_n w^2\mathbf{w}&A\rho_s\rho_n w^2\mathbf{w}&0\end{bmatrix}$$

$$\mathbf{q}=\begin{bmatrix}\phi q-\phi\bar{\bar{\tau}}:\nabla\cdot\mathbf{v}_n-\rho c^2\mathbf{v}_n\nabla\frac{\rho_n}{\rho}-\rho c^2\mathbf{v}_s\nabla\frac{\rho_s}{\rho}-\phi T\mathbf{w}\nabla\rho_s s-\phi w^2\mathbf{w}\nabla\frac{\rho_s\rho_n}{2\rho}\\ -\nabla\cdot\bar{\bar{\tau}}+\rho_n\mathbf{g}\\ \rho_s\mathbf{g}\\ q-\bar{\bar{\tau}}:\nabla\cdot\mathbf{v}-\rho\phi C_v T\mathbf{v}_n\nabla\frac{\rho_n}{\rho}-\rho\phi C_v T\mathbf{v}_s\nabla\frac{\rho_s}{\rho}-T\mathbf{w}\nabla\rho_s s-w^2\mathbf{w}\nabla\frac{\rho_s\rho_n}{2\rho}\end{bmatrix}$$