1) Introduction

As part of the R&D for a Very Large Hadron Collider (100 TeV), high field magnets are being designed at Fermilab. A ten- or more fold increase in synchrotron radiation heat load from the LHC design to a hypothetical VLHC design is a serious issue, which will definitely require separate vacuum and beam-screen studies. However, it would be desirable for the first VLHC magnet prototypes to have a cold-bore, which can match all possible vacuum and heat evacuation systems being brought forward by such studies. We therefore scaled the current state of the art LHC beam screen design to a 50/50 TeV VLHC machine. The aim of the study is to determine if in principle such a beam-screen design could cope with the strong VLHC heat loads. In case the design has to be scaled to the higher heat load, the question of the space requirements of such an upscaled beam-screen design arises.
2) LHC Beam Screen Design

The high-energy proton beams of the Large Hadron Collider (LHC), a superconducting accelerator cooled by superfluid helium at 1.9 K, will induce heat loads into the cryogenic system through several mechanisms:

• synchrotron radiation from bending the beam
• image currents in the resistive wall of the tube (featuring the “anomalous skin-effect”\(^1\))
• inelastic scattering by residual gas molecules
• acceleration of photoemitted electrons by the beam electrical field (“electron cloud instability” or “multipacting”\(^2\)).

These heat loads will be intercepted by an inner shield (beam screen, see figure 1), separated from the beam tube. The double pipe design\(^3\) improves the Carnot efficiency of the cooling scheme because the beam screen is kept at a higher temperature (5 K-20 K) than the magnet cold bore (1.9 K). Furthermore, being at a higher temperature it acts as an intermediate temperature baffle for the cryopump constituted by the 1.9 K surface of the magnet cold bore, thus preventing desorption of the trapped gas molecules and avoiding breakdown of the vacuum. The screen is cooled by forced flow (1-2g/s) supercritical helium (3 bar, 5-20 K) in two nonmagnetic stainless steel tubes, attached to the beam screen on top and bottom. The beam screen design\(^4\) is based on a 1mm thick non-magnetic stainless steel tube with a round cross-section, flattened top and bottom. The space between it and the surrounding 1.9 K cold bore is just sufficient for the cooling pipes. The beam screen is centered within the cold bore by supports. To minimize resistive wall heating the interior surface of the screen is covered by a thin (~50 µm) collaminated (by cold rolling) high purity copper layer. The pumping slots are rounded and randomly distributed to avoid increased beam-

\(\text{Figure 1: The LHC beam screen}^{[3]};\)
coupling impedance and resonance effects. The extruded stainless steel cooling tubes are continuously laser-welded to the beam screen. Intensive R&D is dedicated to: the thermal behavior (cryogenic tests under heat load, flow instabilities), material studies (low susceptibility materials, welding technique, copper inlay technique, insulating support materials), mechanical behavior of the beam screen (twisting, deformation due to weight, contraction during cool-down, reaction to quench pressure), mirror current studies and vacuum issues (gas-desorption, cryopumping).

2a) Heat Load

The LHC heat load data published by Cern[5] are listed in the following table (Table 1):

<table>
<thead>
<tr>
<th>Dep. on E</th>
<th>Dep. on I</th>
<th>Average nominal [W/m]</th>
<th>Average ultimate [W/m]</th>
<th>Peak nominal [W/m]</th>
<th>Peak ultimate [W/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchrotron radiation</td>
<td>E⁴</td>
<td>I</td>
<td>0.164</td>
<td>0.260</td>
<td>0.206</td>
</tr>
<tr>
<td>Resistive wall heating</td>
<td>I²</td>
<td></td>
<td>0.200</td>
<td>0.502</td>
<td>0.200</td>
</tr>
<tr>
<td>Multipacting photoelectrons</td>
<td>E</td>
<td>I³</td>
<td>0.094</td>
<td>0.371</td>
<td>0.200</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>0.458</td>
<td>1.133</td>
<td>0.606</td>
</tr>
</tbody>
</table>

Table 1: Estimation of heat load into the LHC beam screen[5]. Nominal conditions refer to \( E=7 \text{ TeV}, I=536 \text{ mA} \), ultimate conditions are: \( E=7 \text{ TeV}, I=848 \text{ mA} \).

The maximum permissible average heat load per beam is approximately 1 W/m for the LHC design. Multipacting has recently been identified as a serious problem, which may result in heat loads exceeding the design limit.

3) Scaling the LHC Beam Screen to the VLHC Heat Load

3a) Estimated heat loads of a VLHC configuration

The VLHC heat load calculations are based on the following machine characteristics:

| Energy per proton \( E \) | 50 TeV |
| Peak Luminosity \( L \) | \( 10^{34} \text{ cm}^{-2}\text{s}^{-1} \) |
| Circumference \( C \) | 89 km |
| Dipole Field \( B \) | 12.5 T |
| Number of Bunches | 20000 |
| Initial Nr. of Protons per Bunch | \( 1.25\times10^{10} \) |
| Number of protons per beam \( N_{\text{prot}} \) | \( 2.5\times10^{14} \) |
| Bunch Spacing | 4.45m |
| Beam Current \( (N_{\text{prot}}/C) \) | 0.135A |

Table 2: VLHC machine parameters[6]

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The VLHC heat loads can be scaled to the LHC values through their dependence on particle energy, circumference (or magnetic field) and beam current using the following scaling laws for:

- the synchrotron radiation heat load

\[
\frac{p_{VLHC}^{SR}}{p_{LHC}^{SR}} = \left(\frac{E_{VLHC}}{E_{LHC}}\right)^4 \left(\frac{R_{LHC}}{R_{VLHC}}\right)^2 \left(\frac{I_{VLHC}}{I_{LHC}}\right),
\]

- the resistive wall heating heat load (no anomalous skin effect included and assuming that the bunch length is equal in the LHC and the VLHC configuration)

\[
\frac{p_{VLHC}^{RW}}{p_{LHC}^{RW}} \approx \left(\frac{I_{VLHC}}{I_{LHC}}\right)^2,
\]

- and the photoelectron contribution (assuming that the photoelectron yield is not increased by the higher critical photon energy in the VLHC and with some uncertainty regarding the energy dependence - which could be as well \(\sim E^2\) or \(\sim E^4\)).

\[
\left(\frac{p_{VLHC}^{PE}}{p_{LHC}^{PE}}\right) = \left(\frac{E_{VLHC}}{E_{LHC}}\right)^\left(\frac{I_{VLHC}}{I_{LHC}}\right)^3 \left(\frac{R_{LHC}}{R_{VLHC}}\right)^2.
\]

Using these scaling laws combined with the machine/beam parameters in above table, the expected heat loads in a VLHC machine can be computed.

<table>
<thead>
<tr>
<th></th>
<th>Dep. on E</th>
<th>Dep. on R</th>
<th>Dep. on I</th>
<th>Average nominal [W/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Synchrotron radiation</strong></td>
<td>(E^4)</td>
<td>(1/R^2)</td>
<td>(I)</td>
<td>7.8</td>
</tr>
<tr>
<td><strong>Resistive wall heating</strong></td>
<td>(I^2)</td>
<td></td>
<td></td>
<td>0.015</td>
</tr>
<tr>
<td><strong>Multipacting photoelectrons</strong></td>
<td>(E)</td>
<td>(1/R^2)</td>
<td>(I^3)</td>
<td>0.01</td>
</tr>
</tbody>
</table>

*Table 3: Heat loads on a hypothetical VLHC LHC-type beam screen. Values are scaled from the average nominal LHC conditions in Table 1. The uncertainty concerning the photoelectron contribution is big.*

The synchrotron radiation is clearly the dominant source of heat load. Eventually it is feared that multipacting could become as well a dominant source of heat load in a VLHC type machine. Furthermore a design based on a higher luminosity than \(10^{-34}\) cm\(^{-2}\)s\(^{-1}\) will result in an even higher synchrotron radiation heat load.

### 3b) Cooling the VLHC beam screen

The following describes the simplified steady state thermal model we use to describe the beam screen cooling. A chain of thermal resistances represents the heat path from the point where the synchrotron radiation hits the screen to the cryogenic system. The parameters dominating the temperature profile across the heat path is the thermal
impedance of the steel between the copper on the inside of the beam screen and the cooling tube, and the forced convection heat transfer in the flowing helium (Figure 2). It is supposed that all other contributions (variations in the cross-section of the heat path, the thermal impedance of the thin copper layer on the inside of the screen, the boundary layer Kapitza resistance) to the total temperature drop are negligible. Additionally, it has to be verified that the cooling capacity of the forced flow supercritical helium volume always matches the heating power coming from the beam screen. We assumed that a unit length of the cryogenic system cooling the beam screen is 200 m. The helium temperature will vary along this length between $T_{in}$ and $T_{out}$. Superimposed to the longitudinal temperature profile there will be a radial temperature profile, which is the driving force of the convective heat transfer into the cryogen. We imposed that the longitudinal temperature difference in the cryogenic system along the 200 m should remain smaller or of the order of 30 K. Furthermore, we imposed that the pressure drop along this length should remain within 25% of the system pressure. Presumably, the strongest simplification lies within the fact, that we calculated the solutions at a point along the line where the coolant temperature is the average of $T_{in}$ and $T_{out}$. The stronger the longitudinal temperature difference in the system, the less valid is the model for the extremities of the cooling circuit.

![Figure 2: Simplified thermal network model for the beam screen cooling](image)

*Figure 2: Simplified thermal network model for the beam screen cooling: The incoming heating power $P_{bs}$ is conducted through the steel path ($R_{SS}$), transferred into the helium through forced convection ($R_{He}$) and finally transported in the helium $P_{He}$. The key temperatures along the path are $T_{bs}$, the beam screen temperature, $T_{ct}$, the inner wall temperature of the cooling tube (and approximately the radial peak temperature of the helium) and $T_{0}$ the average helium temperature along the cryogenic system.*

**3c) The “Optimum” Beam Screen Temperature**

Prior to the calculation of the required VLHC beam screen cooling capacity, the thermodynamic optimum for the beam-screen or coolant temperature at the given rate of heat deposition was calculated. Therefore, the power per unit length of beam tube was computed for beam screen and beam tube (considering conductive and radiative heat exchange) and both functions multiplied by a Carnot-efficiency factor taking into account
the temperature of the screen and the tube. The cold bore (cb) temperature $T_{cb}$ was set to 4.2 K, the beam screen temperature $T_{bs}$ was chosen to be the free variable.

\[
\frac{dp_{tot}(T_{bs})}{dT} = 0 \quad \quad p_{tot} = p_{bs}(T_{bs})f(T) + p_{cb}(T_{cb})f(T_{cb}) \\
\quad f(T) = \frac{1}{T} \left( \frac{T}{T_R - T} \right) \eta
\]

\[
p_{cb}(T_{bs}) = C_1 (T_{bs}^{2.34} - T_{cb}^{2.34}) + C_2 \sigma (T_{bs}^{4} - T_{cb}^{4}) \quad \left[ \frac{W}{m} \right] \\
p_{bs}(T_{bs}) = p_0 - p_{cb}(T_{bs})
\]

The powers $p$ are in terms of Watts per meter of beam tube, $T_R$ is the room temperature, $\eta$ the efficiency of the refrigerator (assumed to be 0.3), $f(T)$ the Carnot-efficiency for refrigerators, $C_1$ a geometrical constant ($2.4 \times 10^{-6} \text{ W/m/K}^{2.34}$) as measured at Cern for the supportless ("touching") beam screen design[^7], $C_2$ a geometrical function containing the emissivity of two parallel surfaces. $p_0$ is the expected heat load on the beam screen (see Table 3). $P_{tot}$ as a function of beam screen temperature is shown in the next plot. In the calculation of the Carnot efficiency of the beam screen cooling the initial temperature of the cryogenic fluid was assumed to be $T_{bs}$ - 20K.

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Figure 3: Estimation of the total cooling power (at room temperature) versus beam screen temperature (the cold bore temperature was fixed to 4.2 K) for a projected VLHC beam screen heat load of 8 W/m.
The thermodynamic optimum temperature is at a broad minimum around 100 K. Obviously the temperature of the coolant (which is in fact the relevant temperature at which the power extraction occurs) is different from the beam screen temperature. Since the beam-screen temperature and the helium temperature are coupled (see Figure 2), we are not free to choose them independently. Therefore it is required that the beam screen temperature and the helium temperature $T_0$ are both within the broad minimum of the $P_{\text{tot}}$ curve. The typical temperature difference $T_{\text{bs}} - T_0$ is roughly 20 K. Another aspect of this calculation is that the total cold-bore heat load is mainly generated in the magnets (through beam- and ramping loss) or through static heat leaks into the cryostat. The heat transferred from the beam screen to the cold bore is usually only a fraction of that heat load (typically in the order of 4 mW/m). Assuming an additional 0.2 W/m cold bore heat load coming from the magnet operation adds an offset of 50 W/m to $P_{\text{tot}}$ in Figure 3.

To keep the resistive wall heating load contribution small we rejected solutions with $T_{\text{bs}} > 50$ K (which is indeed an arbitrary number). Taking into account a typical temperature difference of 20 K between the beam-screen and the incoming coolant, the beam-screen cooling will cost roughly 2.1 kW/m ($\sim$200 MW over the total ring) of plug in refrigeration power. This figure corresponds to the double of the total refrigeration power need of the LHC. Although some solutions of the cooling problem will be proposed in the following, they will not remove this issue of the tremendous cost of such a VLHC heat load at low beam screen temperatures.

3d) Results of the Modeling

To solve the cooling problem we sought a common solution to the basic set of equations mentioned above: a continuity equation for the heat flux from the spot hit by radiation to the cooling system and a global energy balance equation. The heat flux equation demands that the heat flux from the hot spot to the cooling tube $Q_{\text{cond}}$ and the flux being transferred into the helium $Q_{\text{conv}}$ meet at the incoming level of heat flux given by the beam screen heating power $P_{\text{bs}}$.

The conductive heat transfer in the beam screen is of the Fourier-type, using an integrated stainless steel thermal conductivity ($k_{\text{SS}}$) and a geometrical factor ($S$ is the cross-sectional area of the heat path over the full length of the beam screen cooling system), $L$ the length of the beam screen cooling system (200 m) and $l$ the length of the heat path).

$$Q_{\text{cond}} = \frac{S}{IL} \int T_0 k_{\text{SS}}(T) dT$$

The cooling heat transfer-correlation is of the forced convection type – with the helium in the supercritical state. It is assumed that the flow is turbulent. The average helium temperature $T_0$ and the peak temperature (assumed to be the wall temperature $T_{\text{ct}}$) are variables. $N$ is the number of LHC-type cooling pipes (design: 4.5 mm od, 0.4 mm wall thickness), $A_{\text{ct}}$ the inner cooling tube wall surface over the full length $L$ of the cooling system, the heat transfer coefficient $h$ calculated from the Nusselt-number at the initial helium temperature and the helium heat conductivity $k$. The contributors to the Nusselt number are the density $\rho$, the viscosity $\mu$, the flow velocity $v$, the hydraulic diameter of
the cooling tube (which is the inner diameter $d_{ct}$), the specific heat $c_p$ and the heat conductivity $k$.

$$\nu(T) = \frac{m}{A \cdot \rho(T) \cdot N} \left[ \frac{m}{s} \right]$$

$$Q_{\text{conv}} = N \cdot \frac{A_{ct}}{L} h(T_0)(T_{ct} - T_0) \left[ \frac{W}{m} \right]$$

$$h(T_0) = Nu \frac{k(T_0)}{d_{ct}}$$

$$Nu = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4}$$

$$\text{Re} = \rho(T_0) \nu \cdot \frac{d_{ct}}{\mu(T_0)}$$

$$\text{Pr} = \mu(T_0) \frac{c_p(T_0)}{k(T_0)}$$

The energy balance equation requires that the heat deposit on the beam screen has to be at least equal to the heat removal capacity of the helium. The heat removal capacity of the helium is calculated from the mass flow rate $dm/dt$, the temperature difference over the full length of the cryogenic system $T_{\text{out}} - T_{\text{in}}$, and the specific heat $c_p$, taken at the average longitudinal helium temperature in the system ($T_0$).

$$p_{\text{He}} = \frac{dm}{dt} c_p(T_0)(T_{\text{out}} - T_{\text{in}}) \frac{1}{L} \left[ \frac{W}{m} \right]$$

The solution is found iteratively from a first guess of $N$, $dm/dt$, $T_{\text{out}}$, $T_{\text{in}}$, $T_{ct}$ and $T_{bs}$ by satisfying the following conditions:

- $Q_{\text{conv}}(T_{ct})=Q_{\text{cond}}(T_{ct})=p_{bs} < p_{\text{He}}$ with $T_{ct}$ being in agreement with its input value
- $T_{bs} > T_{ct} > T_{out}$
- $(T_{\text{in}} - T_{\text{out}}) = L \cdot \frac{p_{bs}}{c_p(T_0)} \frac{dm}{dt}$
- $(T_{\text{in}} - T_{\text{out}}) \leq 200 \frac{mK}{m}$
- pressure drop along the 200 m pipe is smaller than 25% of the system pressure
- $T_{bs} < 50$ K
- $T_{\text{in}}$ as high as possible to improve thermodynamic efficiency

The geometrical parameters are kept as in the LHC design. A plot showing a possible solution is shown in Figure 4.
The list of solutions in Table 5 (A-D) is not complete. Only a few solutions are technologically feasible. It turns out that the most critical part of the operation is the heat absorption capacity of the cryogenic system.

**Figure 4:** A possible solution for the VLHC beam screen cooling (D in Table 5). Heat conduction along beam screen $p_{\text{cond}}$, forced convection heat transfer to helium $p_{\text{conv}}$, Heat load on beam screen $p_{bs}$ and heat transport capacity of the helium $p_{He}$, all in W/m, as a function of the cooling tube wall temperature.

The list of solutions in *Table 5* (A-D) is not complete. Only a few solutions are technologically feasible. It turns out that the most critical part of the operation is the heat absorption capacity of the cryogenic system.

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>14</td>
<td>66</td>
<td>40.2</td>
<td>40.1</td>
<td>3</td>
<td>750</td>
<td>6</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>38</td>
<td>62</td>
<td>50.1</td>
<td>50.04</td>
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<td>C</td>
<td>10</td>
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<td>22.08</td>
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<td>926</td>
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<td>70</td>
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<td>20</td>
<td>2000</td>
<td>6</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5:** Possible solutions to the VLHC beam-screen cooling problem. $T_{in}$ is the helium entry temperature, $T_{out}$ is the helium exit temperature (refering to the beam screen cryogenic system of length $L=200$ m), $T_{bs}$ is the beam screen temperature, $T_{ct}$ is the wall temperature of the cooling tube, $T_0$ is the average helium temperature in the center of the tubes, $P$ and $\Delta p$ are the pressure and the pressure drop in the 200 m cryogenic system, $dm/dt$ is the helium flow rate, $N$ the number of cooling tubes (refering to LHC type cooling tubes with id=3.7 mm). * The first row refers to the LHC design (with a heat load of <1 W/m). A is a “baseline”-solution. B, C and D are optimized for high $T_{in}$ (B), low beam screen temperature (C) and a small number of cooling tubes (D). All “optimized” solutions reach the technological limits in the other parameters.
An excessive rise in helium temperature (limit ~0.2 K/m) has to be avoided as well as a strong pressure drop. The cooling capacity of the system can be improved in many ways: by increasing the number of cooling tubes, lowering the entry temperature of the helium, increasing the flow rate or increasing the operating pressure (to keep the helium density high). All solutions in Table 5 operate at the limit of technological feasibility: either very high flow rates exceeding 10 g/s, a big number of channels (>4), low helium temperatures or high beam screen temperatures are required. Other possibilities to improve the heat transfer correlation have not been considered here (e.g. the use of liquid hydrogen as cryogen). The use of superfluid helium, although tempting because of improved heat transfer correlation, is not reasonable because of the low Carnot-efficiency at the temperatures below $T_\lambda$.

### 3d) Space Requirements for a VLHC Beam Screen

The baseline design (A), presented in Table 5, operates at a rather low $T_{in}$, a relatively high $T_{bs}$ and still needs 8 LHC-type cooling tubes to cope with the synchrotronic heat load. Any optimization with respect to thermodynamic efficiency (increasing $T_{in}$) or decreasing $T_{bs}$ immediately requires a better cooling performance. Therefore it is advisable to project a number of tubes of 10 or even higher. Roughly estimating the space requirement for one LHC-type cooling tube to ~20 mm$^2$, such that 10 tubes would require at least 200 mm$^2$. The extruded cooling pipes (4.5 mm od) are considered as the limit of technology and therefore any further miniaturization has not been considered. The support-less LHC beam screen design (49 mm cold bore id) has a total of 1900 mm$^2$, with 150 mm$^2$ being available on top and bottom. The 2 times 4 cooling tubes could fit into these cavities. As well the 15-tube solution B could fit. A similar calculation for a hypothetical 45mm id cold bore magnet (36 mm vertical beam-screen dimension) reveals that the cavity has 83 mm$^2$, which can at best case house 2 times 4 tubes. It is not clear to which extent the filling of this cavity interferes with the cryopump-function. However, it is advisable to use only one duct having the shape of the cavity between beam screen and cold bore. The more efficient use of space could make a 10 tube 45 mm bore design possible. Furthermore, good surface to volume ratios can be achieved this way. The only way to operate a LHC-type beam screen in a magnet with a smaller bore (<50 mm) is through the use of supercritical helium at higher pressure (see solution D). The hoop stress in a 20 bar LHC-type cooling tube would be ~20 MPa, which is tolerable.

![Figure 6: Type A (table 5) beam screen design: 8 tubes (3.7 mm id / 4.5 mm od), screen (44 mm id / 46 mm od) and cold bore (49 mm id / 52 mm od);](image-url)